MPS115/116: HOMEWORK 4

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1. Lagrange Interpolation

Lagrange interpolation is a method used to fit smooth polynomial curves to sets of points in the plane. Suppose we have n distinct points in the plane, where n is an integer greater than 1, with no two x-coordinates equal. Then there is a polynomial $f(x)$ known as the *Lagrange polynomial* of degree at most $n-1$ which passes through the points. In some sense the Lagrange polynomial is the simplest smooth curve that fits the points.

2. A quadratic interpolation of $\sin(x)$

We can get a quadratic interpolation of $\sin(x)$ in the following way. The points $(0,0)$, $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{3\pi}{4}, \frac{1}{\sqrt{2}}\right)$ all lie on the curve $y = \sin(x)$. Substituting these points in to the formula

$$f(x) = \sum_{i=1}^{3} \left(\prod_{\substack{j=1 \\ j \neq i}}^{3} \frac{x - x_j}{x_i - x_j} \right) y_i$$

we obtain

$$f(x) = \frac{x(x - 3\pi/4)}{\pi/4(\pi/4 - 3\pi/4)} \frac{1}{\sqrt{2}} + \frac{x(x - \pi/4)}{3\pi/4(3\pi/4 - \pi/4)} \frac{1}{\sqrt{2}}.$$

The expression simplifies to $f(x) = \frac{16}{3\pi^2\sqrt{2}} x(\pi - x)$. This is our quadratic interpolation of $\sin(x)$. You can see how well $f(x)$ approximates $\sin(x)$ by looking at the graph in Figure 1.

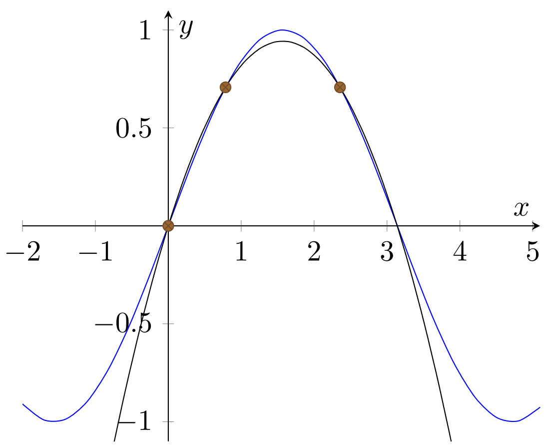

Figure 1. The graphs of $y = \frac{16}{3\pi^2\sqrt{2}} x(\pi - x)$ and $y = \sin x$

3. References

- Wikipedia contributors, *Lagrange polynomial*, Wikipedia, The Free Encyclopedia. Visited 26 October 2012, updated 25 October 2012, http://en.wikipedia.org/wiki/Lagrange_polynomial.
- K. A. Stroud, Advanced engineering mathematics. Palgrave Macmillan, 2011.