MAS116/MAS117 Presentation Lecture 1: Communicating mathematics and a first look at LaTeX

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> *We should not write so that it is possible for our readers to understand us, rather we should write so that it is impossible for them to misunderstand us.*
>
> De Institutione Oratoria, Book VIII, 2, 24

1 Introduction to this part of the module

The theme for this part of the module

The theme for the lectures and labs on a Tuesday this semester is *the written communication of mathematics.* This involves two separate aspects:

(i) learning appropriate software to typeset mathematical documents (LaTeX and HTML);

(ii) improving your mathematical writing skills in English.

Note.

- These will be important in writing the reports for the coursework.
- Writing clearly ties in with the skills that your tutor will be trying to help you improve in the core mathematics module.
- It will remain important throughout your degree, particularly in exams!
- It will also be an important and desired skill beyond your degree.

The goals and a rough timetable

By the end of the semester, you should

- be better at expressing your ideas;
- be more fluent in your mathematical writing;
- be able to create impressive mathematical webpages and reports.

These will be acheived as follows.

- Up to Reading Week: learning LaTeX and mathematical writing.
- Around Reading Week: a mini-project involving a mathematical investigation and a mathematical report written with LaTeX.
- After Reading Week: creating mathematical webpages with HTML.
- Around Christmas: a group project involving a mathematical investigation written up as a website.

2 Mathematical writing

The need for clarity

If you aren't clear when explaining something face-to-face then people can ask you questions. You can rethink and try to explain more clearly.

If you aren't clear when writing then the reader will just struggle with understanding. So written mathematics needs to be absolutely clear from the outset.

Grammar, punctuation and correct use of symbols are all key tools in making written mathematics clear.

Written mathematics must be readable

When people read mathematics they typically read it out loud in their head. Symbols have specific meanings and specific pronunciation. Change the symbols for words: does what you have make sense?
Example. For all x, $e^x > 0$.
If $e^x(x^2 - 4) > 0$ then $x^2 - 4 > 0$.
Therefore $(x + 2)(x - 2) > 0$, so $x < -2$ or $x > 2$.

This becomes the following when read.

> *For all* x*,* e^x *is greater than* 0*.*
> *If* $e^x(x^2-4)$ *is greater than* 0 *then* x^2-4 *is greater than* 0*.*
> *Therefore,* $(x+2)(x-2)$ *is greater than* 0*, so* x *is less than* -2 *or* x *is greater than* 2*.*

This reads very well.

A poor attempt at writing

Now look at this example.
Example. πr^2

$$r^2 = A/\pi$$
$$\sqrt{A/\pi} \qquad \sqrt{10/\pi}$$
$$\underline{3.18}$$

This becomes the following when read.

> πr^2
> r^2 *is equal to* A/π
> *The square-root of* A/π
> *The square-root of* $10/\pi$
> 3.18

This is dreadful!

Activity time

Activity. Introduce yourself to the person next to you (or talk in a group of three). For the bad solution above, try to work out what question was asked, then each write a much better solution, correcting errors and improving the solution above as much as possible. Compare what you wrote with your neighbour, and discuss.

A first solution to the activity

That example was a bad attempt at answering the following.

> *'Write down an expression for the radius* r *of a circle of area* A*. Use it to find the radius when the area is* $10\,\mathrm{cm}^2$*.'*

Here's a reasonable solution to the problem.

> *The area is given by* $A = \pi r^2$*.*
> *Therefore* $r = \pm\sqrt{A/\pi} = \sqrt{A/\pi}$*, since* $r > 0$*.*
> *If* $A = 10$ *then* $r = \sqrt{10/\pi} \approx 1.78$*.*
> *Thus the radius is* $1.78\,\mathrm{cm}$ *(2 d.p.).*

This is better than the original attempt. This is hopefully close to what you'd submit as a handwritten solution.

Some comments on this first solution

Note that

- where we've used symbols, the result still reads as full sentences (including full stops!);
- we've explained clearly what is being calculated when;
- the answer no longer 'floats' without being tied to anything;
- the answer is calculated precisely as $\sqrt{\frac{10}{\pi}}$, then as an approximation with a stated accuracy of 2 decimal places.

Notice the use of the '$\approx$' sign!

A more formal rewrite of the solution

> *The area A for a circle with radius r is given by $A = \pi r^2$. Thus $r^2 = \frac{A}{\pi}$ and so $r = \sqrt{A/\pi}$, since $r > 0$. When $A = 10$ this gives $r = \sqrt{10/\pi}$. Hence, when the area of the circle is $10\,\mathrm{cm}^2$ its radius is $\sqrt{10/\pi}\,\mathrm{cm} \approx 1.78\,\mathrm{cm}$.*

This has quite a different tone.

While both of the solutions above are well-written, they serve different purposes.

The first one is well-judged for a *handwritten* homework.

The second solution uses fewer symbols and fewer line-breaks. It is the way mathematics is written in books, papers and reports. This is the style that we will be focusing on.

3 Four basic rules of writing mathematics

Four basic rules

1. Do use proper grammar, and write in full sentences.
2. Don't start a sentence with a symbol.
3. Don't use too many symbols; often, words are better.
4. Do use symbols correctly and don't invent your own.

1. Do use proper grammar and write in full sentences.

Maths is hard enough already without making the English hard! This includes putting full-stops after formulae if they're at the end of the sentence, as in

> *Hence we see that*
> $$f(x) = \cos x.$$

Sometimes the sentence carries on, as in

> *With f given by*
> $$f(x) = \cos x,$$
> *it is clear that...*

in which case use a comma, or nothing at all.

2. Don't start a sentence with a symbol.

It makes it hard to spot the start of the sentence. There's a trick for this. For example, the sentence

> *ϕ is continuous.*

can be rewritten as

> *The function ϕ is continuous.*

3. Don't use too many symbols; often, words are better.

For example, avoid using the symbols $\therefore$, $\because$, $\implies$ and $\iff$ in typeset mathematics.

In this course,

> *Now, $x > 0 \implies x^3 + x > 0$. $\therefore f(x) = x^3 + x > 0$ for all $x \in \mathbb{R}^+$.*

is not as appropriate as

> *Now, if $x > 0$ then $x^3 + x > 0$. Thus, the function $f(x) = x^3 + x$ is positive for all positive real numbers x.*

Note that the three dots are more for high-school maths than university maths; and the arrows have formal logical meaning.

4. Do use symbols correctly and don't invent your own.

Each mathematical symbol has a precise meaning.

People sometimes use vague arrows such as $\rightsquigarrow$, but what does this actually mean?

Summary

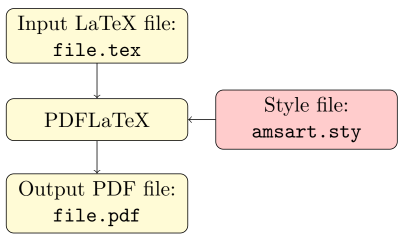
Good mathematical presentation is very important. It helps to create clear, readable solutions. It also helps you to think clearly.

4 A first look at LaTeX

LaTeX is a mark-up language

In word processing software like Word or Google Docs you edit the output document. This is called 'what you see is what you get' (WYSIWYG) editing.

LaTeX uses so-called 'mark-up' editing (as does HTML). You prepare a text file with instructions about the structure of the document. This is fed into the PDFLaTeX program which combines this with style information to produce a PDF document.



An example

[See the lab.]

You will see this in action in the lab this afternoon.